Title	The classification of association schemes with 13 or 15 points
Author(s)	Hirasaka, Mitsugu; Suga, Yuji
Citation	数理解析研究所講究録 (1996), 962: 71-80
Issue Date	1996-08
URL	http://hdl.handle.net/2433/60543
Right	
Type	Departmental Bulletin Paper
Textversion	publisher

数理解析研究所講究録
962巻 1996年 71-80

The classification of association schemes with 13 or 15 points

Mitsugu Hirasaka and Yuji Suga
平坂 貢 須賀 祐治

Graduate School of Mathematics , Kyushu University
数理学研究科 九州大学

Abstract

Association schemes are deeply related to codes, designs, finite geometry and groups. We would like to classify association schemes in general, but it is too difficult. So we restrict the cardinality of X $(:= |X|)$.

Association schemes of following cardinalities are already classified.

• $|X| \leq 10$ (Nomiyama[4]) • $|X| = 11,12$ (Hirasaka[6]) • $|X| = 14$ (Sakita[5])

We classify association schemes with 13 or 15 points in Section 2. In Section 3, we introduce a directed conference graph related to 2-designs and Hadamard matrices. In Section 4, we introduce Johnson schemes and observe fission schemes of theirs which are not group cases.

1 Introduction

Definition 1 (Association scheme) *Let X be a finite set of cardinality n. Let $R_i (i = 0, \ldots, d)$ be subsets of $X \times X$. The configuration $\mathcal{X} = (X, \{R_i\}_{i=0,\ldots,d})$ is called an association scheme if it satisfies the following properties(i),...,(iv).*
(i) $R_0 = \{(x,x) | x \in X\}$.
(ii) $R_0 \cup R_1 \cup \ldots \cup R_d = X \times X$, $R_i \cap R_j = \phi$ *if* $i \neq j$.
(iii)$R_i^t = R_{i'}$ *for some* $i' \in \{0, 1, \ldots, d\}$ *,where* $R_i^t := \{(y,x) | (x,y) \in R_i\}$.
(iv) For $i, j, k \in \{0, 1, \ldots, d\}$ *, the number of* $\{z \in X | (x,z) \in R_i, (z,x) \in R_j\}$ *is constant whenever* $(x,y) \in R_k$. *This constant is denoted by* p_{ij}^k.

An association scheme is called commutative if it satisfies the property (v) and called symmetric if it satisfies the property (vi) .
(v) $p_{ij}^k = p_{ji}^k$ *for* $\forall\ i, j, k$.
(vi) $R_i^t = R_i$ *for* $\forall\ i$.

Let $k_i = p_{ii'}^0$. The positive integer k_i is called the valency of R_i.

The i-th adjacency matrix A_i of $\mathcal{X}$ is defined to be the matrix of degree $|X|$ whose rows and columns are indexed by the elements of X and whose (x, y) entries are

$$(A_i)_{xy} = \begin{cases} 1 & \text{if } (x,y) \in R_i \\ 0 & \text{otherwise} \end{cases}$$

It is easy to see that the properties $(i), \ldots, (vi)$ are equivalent to the following $(i)', \ldots, (vi)'$ respectively.
$(i)'$ $A_0 = I$, where I is the identity matrix.
$(ii)'$ $A_0 + A_1 + \cdots + A_d = J$, where J is the matrix whose entries are all 1.
$(iii)'$ ${}^tA_i = A_{i'}$ for some i', where ${}^tA_i$ denotes the transpose of A_i.
$(iv)'$ $A_iA_j = \sum_{k=0}^{d} p_{ij}^k A_k$ for $\forall\ i, j$.
$(v)'$ $A_iA_j = A_jA_i$ for $\forall\ i, j$.
$(vi)'$ ${}^tA_i = A_i$ for $\forall\ i$.

Example 1 (Cyclotomic scheme) *Let q be a prime power, let d be a divisor of q-1 and let g be a primitive root in $\boldsymbol{F}_q$. Then $\boldsymbol{F}_q^{\times} = < g^d > \cup\, g < g^d > \cup \ldots \cup\, g^{d-1} < g^d >$.*
$\mathcal{X} = (X, \{R_i\}_{i=0,\ldots,d})$ is called a cyclotomic scheme if $X = \boldsymbol{F}_q$ with relations $R_0 = \{(x,x) | x \in X\}$ and $R_i = \{(x,y) |\ y - x \in g^i < g^d >\}$ $(1 \le i \le d)$.

Definition 2 (Isomorphic) *Let $\mathcal{X} = (X, \{R_i\}_{0 \le i \le d})$ and $\mathcal{Y} = (Y, \{S_i\}_{0 \le i \le e})$ be association schemes. We say that $\mathcal{X}$ is isomorphic to $\mathcal{Y}$ if there exists the following pair (σ, φ)*

$$\begin{array}{rccc} \sigma: & \{0,1,\ldots,d\} & \longrightarrow & \{0,1,\ldots,e\} \\ \varphi: & X & \longrightarrow & Y \end{array}$$

where σ and φ are bijections such that

$$(\varphi(x), \varphi(y)) \in S_{\sigma(i)} \quad \text{if} \quad (x,y) \in R_i \text{ for } \forall\ x, y \in X$$

Definition 3 (Fusion and Fission) *Let $\mathcal{X} = (X, \{R_i\}_{0 \le i \le d})$ be an association scheme. Let Λ_i $(i = 0, 1, \ldots, e)$ be subsets of $\{0, 1, \ldots, d\}$ with properties that $\Lambda_0 = \{0\}$, $\Lambda_0 \cup \Lambda_1 \cup \ldots \cup \Lambda_e = \{0, 1, \ldots, d\}$ and $\Lambda_i \cap \Lambda_j = \phi$ if $i \neq j$. Define $\tilde{R}_{\Lambda_j}$ to be $\bigcup_{k \in \Lambda_j} R_k$ for $\forall$ $j \in \{0, 1, \ldots, e\}$.*
If $\tilde{\mathcal{X}} = (X, \{\tilde{R}_{\Lambda_j}\}_{0 \le j \le e})$ becomes an association scheme then we say that $\tilde{\mathcal{X}}$ is a fusion scheme of $\mathcal{X}$, and $\mathcal{X}$ is a fission scheme of $\tilde{\mathcal{X}}$. We denote this by
$\tilde{\mathcal{X}}$: $\{1_1 + 1_2 + \ldots + 1_{m_1}, \ldots, e_1 + e_2 + \ldots + e_{m_e}\}$ of $\mathcal{X}$
where $\Lambda_i = \{i_1, i_2, \ldots, i_{m_i}\}$ and $m_i = |\Lambda_i|$.

Example 2 (Group case) *Define a group case scheme be isomorphic to an association scheme whose adjacency matrices are given by a regular representation of some group of order $|X|$. Note that each valency of this scheme is 1.*

Definition 4 (Circulant) *An association scheme $\mathcal{X} = (X, \{R_i\}_{0 \le i \le d})$ is called circulant if $\mathcal{X}$ is isomorphic to a fusion scheme of a cyclic group case of order $|X|$.*

2 Results

Methods *First, we assume that an association scheme $\mathcal{X}$ has fixed valencies. We choose admissible p_{ij}^k's (intersection numbers) by using*

$$\textbf{Def.1 } (iv)' \qquad A_iA_j = \sum_{k=0}^{d} p_{ij}^k A_k \text{ for } \forall\, i,j\,.$$

and decide whether we can construct the adjacency matrices with those intersection numbers or not, then classify association schemes up to isomorphism.

Result 1 (In the case of $|X|$=13)

There are only cyclotomic schemes. There is a one to one correspondence between factors of 12 (=$|X|$-1) and association schemes which have these factor classes. But in general case (for example $|X|$=19, 23) counter-examples exist.(E.Spence[7])

```
d=12   [ 1] cyclic group case (Z13)
   6   [ 2] {1+12,2+11,3+10,4+9,5+8,6+7} of Z13
   4   [ 3] {1+3+9,2+5+6,4+10+12,7+8+11}
   3   [ 4] {1+5+8+12,2+3+10+11,4+6+7+9}
   2   [ 5] {1+3+4+9+10+12,2+5+6+7+8+11}
   1   [ 6] Trivial scheme
```

Remark 1

A translation association scheme with a prime points is a cyclotomic scheme.([2] p.66)

Result 2 (In the case of $|X|$=15)

We obtain 25 non-isomorphic association schemes.

- *21 Circulant schemes*
- *Only one conference graph type scheme(see* **22***) ↪Section 3*
- *Three other type schemes(see* **10,18,21***) ↪Section 4*

```
d=14   [ 1] cyclic group case (Z15)
   9   [ 2] {3,6,9,12,5+10,8+13,1+11,14+4,2+7} of Z15
   8   [ 3] {5,10,3+12,6+9,1+4,11+14,2+8,7+13}
   7   [ 4] {1+14,2+13,3+12,4+11,5+10,6+9,7+8}
   6   [ 5] {3,6,9,12,2+5+8+11+14,1+4+7+10+13}
       [ 6] {5,10,3+8+13,1+6+11,4+9+14,2+7+12}
   5   [ 7] {3,6,9,12,1+2+4+5+7+8+10+11+13+14}
       [ 8] {5,10,3+6+9+12,2+8+11+14,1+4+7+13}
       [ 9] {5+10,3+8+13,1+6+11,4+9+14,2+7+12}
       [ A] {5+10,3+12,6+9,1+4+11+14,2+7+8+13}
       [10] Non circulant scheme
   4   [11] {3+12,6+9,2+5+8+11+14,1+4+7+10+13}
       [12] {5,10,2+3+7+8+12+13,1+4+6+9+11+14}
       [ B] {5+10,3+6+9+12,1+2+4+8,7+11+13+14}
   3   [13] {3+12,6+9,1+2+4+5+7+8+10+11+13+14}
```

[14] {5,10,1+2+3+4+6+7+8+9+11+12+13+14}
[15] {5+10,3+6+9+12,1+2+4+7+8+11+13+14}
[16] {5+10,2+3+7+8+12+13,1+4+6+9+11+14}
[17] {3+6+9+12,2+5+8+11+14,1+4+7+10+13}
[18] Fusion scheme {1+2,3+4,5} of[10]
2 [19] {3+6+9+12,1+2+4+5+7+8+10+11+13+14}
[20] {5+10,1+2+3+4+6+7+8+9+11+12+13+14}
[21] Fusion scheme {1+2+5,3+4} of[10]
[22] Conference graph type
1 [23] Trivial scheme

- *Note that left out [A] and [B] schemes ,on symposium in Kyoto.*

Definition 5 (Relation matrix) *Let $\{A_i\}_{i=0,\dots,d}$ be adjacency matrices of $\mathcal{X}$ $\sum_{k=0}^{d} kA_k$ is called the relation matrix of $\mathcal{X}$.*

The relation matrices are the following.

[1] $\boldsymbol{Z}_n$ cyclic group cases (for n=13,15)

Relation matrix is $\sum_{k=0}^{n-1} k\ {C_n}^k$ where $(C_n)_{ij} = \begin{cases} 1 & \text{if } j \equiv i+1 \text{ (mod n)} \\ 0 & \text{otherwise} \end{cases}$

[10] Non circulant type

$$\begin{bmatrix}
0 & 1 & 2 & 5 & 4 & 3 & 5 & 4 & 3 & 5 & 4 & 3 & 5 & 4 & 3 \\
2 & 0 & 1 & 4 & 3 & 5 & 4 & 3 & 5 & 4 & 3 & 5 & 4 & 3 & 5 \\
1 & 2 & 0 & 3 & 5 & 4 & 3 & 5 & 4 & 3 & 5 & 4 & 3 & 5 & 4 \\
5 & 4 & 3 & 0 & 1 & 2 & 5 & 4 & 3 & 4 & 3 & 5 & 3 & 5 & 4 \\
4 & 3 & 5 & 2 & 0 & 1 & 4 & 3 & 5 & 3 & 5 & 4 & 5 & 4 & 3 \\
3 & 5 & 4 & 1 & 2 & 0 & 3 & 5 & 4 & 5 & 4 & 3 & 4 & 3 & 5 \\
5 & 4 & 3 & 5 & 4 & 3 & 0 & 1 & 2 & 3 & 5 & 4 & 4 & 3 & 5 \\
4 & 3 & 5 & 4 & 3 & 5 & 2 & 0 & 1 & 5 & 4 & 3 & 3 & 5 & 4 \\
3 & 5 & 4 & 3 & 5 & 4 & 1 & 2 & 0 & 4 & 3 & 5 & 5 & 4 & 3 \\
5 & 4 & 3 & 4 & 3 & 5 & 3 & 5 & 4 & 0 & 1 & 2 & 5 & 4 & 3 \\
4 & 3 & 5 & 3 & 5 & 4 & 5 & 4 & 3 & 2 & 0 & 1 & 4 & 3 & 5 \\
3 & 5 & 4 & 5 & 4 & 3 & 4 & 3 & 5 & 1 & 2 & 0 & 3 & 5 & 4 \\
5 & 4 & 3 & 3 & 5 & 4 & 4 & 3 & 5 & 5 & 4 & 3 & 0 & 1 & 2 \\
4 & 3 & 5 & 5 & 4 & 3 & 3 & 5 & 4 & 4 & 3 & 5 & 2 & 0 & 1 \\
3 & 5 & 4 & 4 & 3 & 5 & 5 & 4 & 3 & 3 & 5 & 4 & 1 & 2 & 0
\end{bmatrix}$$

[22] Conference graph type

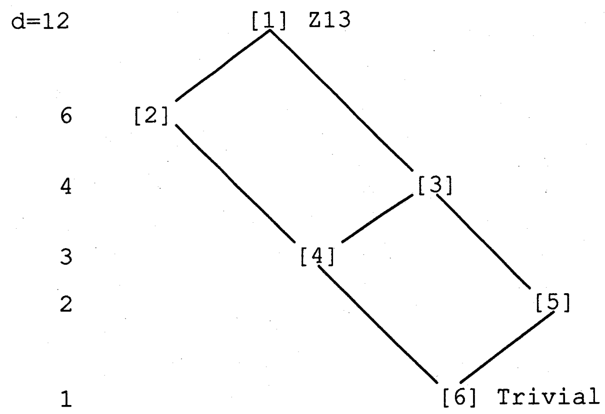
$$\begin{bmatrix}
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
2 & 0 & 1 & 1 & 2 & 1 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & 1 & 2 \\
2 & 2 & 0 & 1 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 2 & 1 & 2 & 1 \\
2 & 2 & 2 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 2 & 1 & 2 & 1 & 1 \\
2 & 1 & 2 & 2 & 0 & 1 & 1 & 2 & 2 & 2 & 1 & 2 & 1 & 1 & 1 \\
2 & 2 & 1 & 2 & 2 & 0 & 1 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 2 \\
2 & 1 & 2 & 1 & 2 & 2 & 0 & 1 & 1 & 2 & 1 & 1 & 1 & 2 & 2 \\
2 & 1 & 1 & 2 & 1 & 2 & 2 & 0 & 2 & 1 & 1 & 1 & 2 & 2 & 1 \\
1 & 2 & 2 & 2 & 1 & 1 & 2 & 1 & 0 & 1 & 1 & 2 & 1 & 2 & 2 \\
1 & 2 & 2 & 1 & 1 & 2 & 1 & 2 & 2 & 0 & 1 & 1 & 2 & 1 & 2 \\
1 & 2 & 1 & 1 & 2 & 1 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 2 & 1 \\
1 & 1 & 1 & 2 & 1 & 2 & 2 & 2 & 1 & 2 & 2 & 0 & 1 & 1 & 2 \\
1 & 1 & 2 & 1 & 2 & 2 & 2 & 1 & 2 & 1 & 2 & 2 & 0 & 1 & 1 \\
1 & 2 & 1 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 1 & 2 & 2 & 0 & 1 \\
1 & 1 & 2 & 2 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 2 & 0
\end{bmatrix}$$

Result 3 (Hesse diagrams)

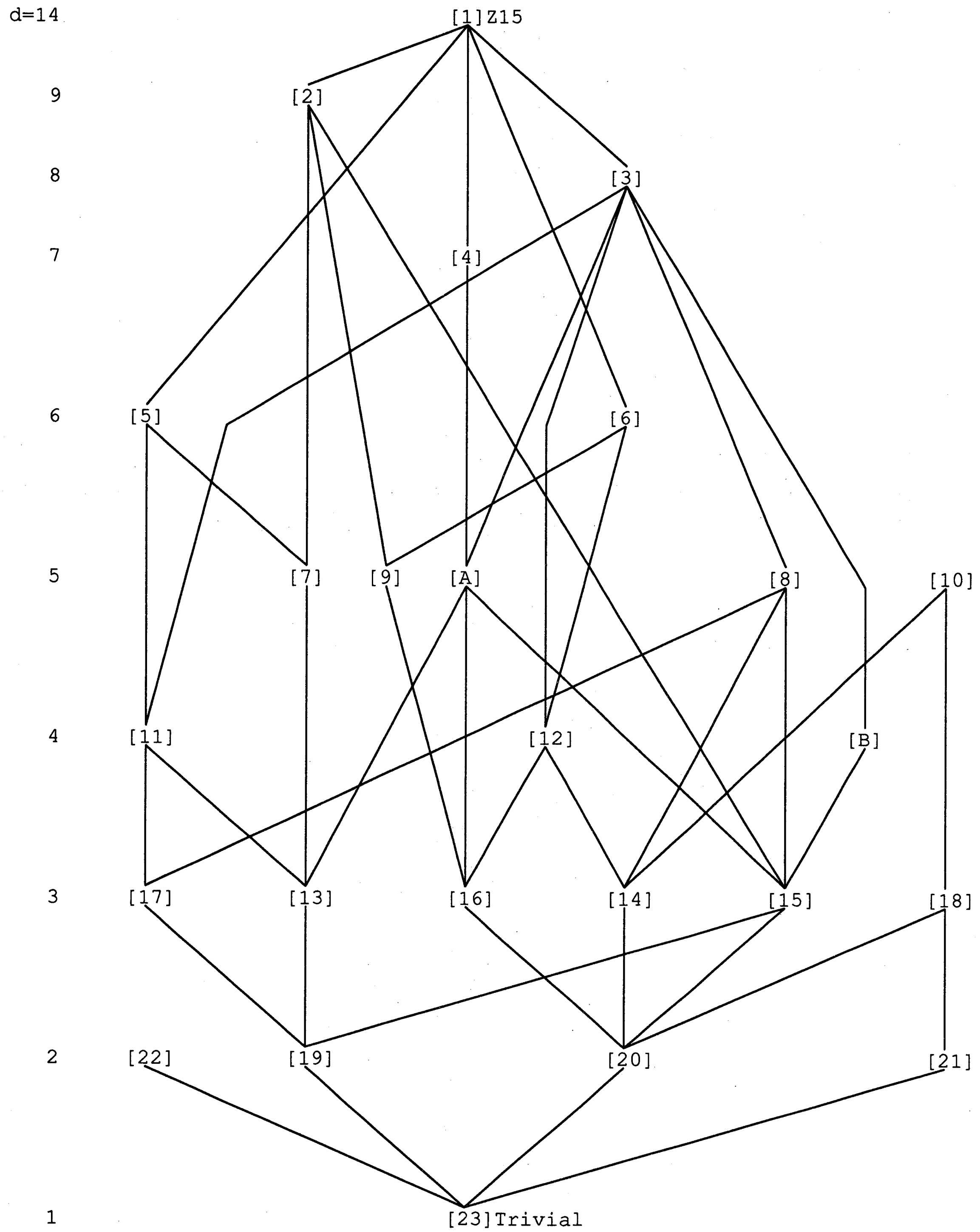
We show the Hesse diagrams for fusion (fission) schemes. Let $\mathcal{X}$ and $\mathcal{Y}$ be association schemes. If $\mathcal{Y}$ is a fusion scheme of $\mathcal{X}$, then we denote this as follows.

$$\mathcal{X} \text{ ———————— } \mathcal{Y}$$

Hesse diagram ($|X|$ =13)



Hesse diagram ($|X| = 15$)



3 Directed conference graphs

Definition 6 (Directed conference graphs) $\Gamma = (V, E)$ *is called a directed conference graph if it satisfies the following properties:*
(0) $(x, y) \in E \iff (y, x) \notin E$ *for* $\forall\ x, y (x \neq y) \in V$.
(i) The number of $\{z \in V | (x, z) \in E\}$ *is constant*$(= k)$.
(ii) The number of $\{z \in V | (x, z) \in E, (z, y) \in E\}$

$$= \begin{cases} \lambda & \text{if } (x, y) \in E \\ \mu & \text{if } (x, y) \notin E \end{cases}$$

Let $\mathcal{X}$ be a non-symmetric association scheme with class 2 and A_i be the adjacency matrices for R_i. Then a graph which has an adjacency matrix A_i (or A_i^t) becomes a directed conference graph. Conversely let A be an adjacency matrix of dir.conf.graph, then a set $\{I, A, A^t\}$ becomes the adjacency matrices of a non-symmetric association scheme. Note that $|X|$ must be 3 (mod 4).

Remark 2
A is an adjacency matrix of a directed conference graph with $|V| = 4m + 3$
$\iff$ *(0)'* $A + A^t = J - I$.
(i)' $AA^t = (m + 1)I + mJ$.
(ii)' $AJ = JA = (2m + 1)J$.

This means that A is an incidence matrix of 2-(4m+3,2m+1,m)design, moreover A is skew. So we find skew incidence matrices of this design.

In this case (m=3), we have 2 non-(design)isomorphic 2-(15,7,3)skewdesigns and we have 2 non-(graph)isomorphic directed conference graphs of $|V|$=15.

Let A be an adjacency matrix of a dir.conf.graph, then $A \not\simeq_\Gamma A^t$. (i.e. A and A^t are representative matrices.) But we have the only one association scheme which has an adjacency matrix A, up to isomorphism of association schemes.

Remark 3
We construct Hadamard matrices of order 16 similar to Paley's constructions. Define

$$\boldsymbol{H} = \left[\begin{array}{c|c} 1 & 1 \cdots 1 \\ \hline -1 & \\ \vdots & \boldsymbol{P} \\ -1 & \end{array}\right] \quad \textit{where } \boldsymbol{P} = 2\boldsymbol{A}_\Gamma + 2\boldsymbol{I} - \boldsymbol{J}$$

A_Γ *:the adjacency matrix of a dir.conf.graph*
I *:the identity matrix*
J *:the all 1 matrix* *of order 15*

Then $\boldsymbol{H}$ becomes a skew Hadamard matrix of order 16. Now let A_Γ be a representative matrix of dir.conf.graph, then we have 2 non-(H-matrix)isomorphic Hadamard matrices. They belong to IV type and V type respectively in Hall's classification.[8]

4 Other type and Johnson schemes

Definition 7 (Johnson schemes) *Let V be a set of cardinality v and let k be a positive integer ($k \leq v/2$). Let X be the collection of k-element subsets of V.*

Relations $\{R_i\}_{i=0,\ldots,d}$ are defined by $(x,y) \in R_i$ $(x,y \in X)$ if and only if $|x \cap y| = k-i$. It is easy to see $\mathcal{X} = (X, \{R_i\}_{0\leq i\leq k})$ is a symmetric association scheme of class k. $\mathcal{X}$ is called the Johnson scheme and denoted by $\boldsymbol{J}(v,k)$.

We observe the fission schemes of $\boldsymbol{J}(6,2)$. In the Hesse diagram of $|X|$=15, we can see that three other type schemes **10**,**18**,**21** and one conference graph type scheme **22** are not fusion schemes of any group case, and **10** is the only one non-commutative scheme with $|X|$=15.

In the case of $|X|$=10, we can observe similar facts. There is a fission scheme of $\boldsymbol{J}(5,2)$ (denoted by $10n_7$ (Nomiyama[4])), and this scheme is non-commutative. Moreover $\boldsymbol{J}(5,2)$ and $10n_7$ are not fusion schemes of any group case.

Remark 4

Note that $\boldsymbol{J}(v,2)$ with $v \geq 5$, $v \not\equiv 7 (\mathrm{mod} 8)$ does not have commutative fission schemes of class 3 .(By Sung Y.Song [9])

Problem 1 *Find and observe fission schemes of $\boldsymbol{J}(v,2)$ $(v \geq 7)$.*

Problem 2 *Find maximal fission schemes except group cases in general.*
(We say that $\mathcal{X}$ is maximal if fission schemes of $\mathcal{X}$ do not exist.)

5 Appendix ($|X|$ =14)

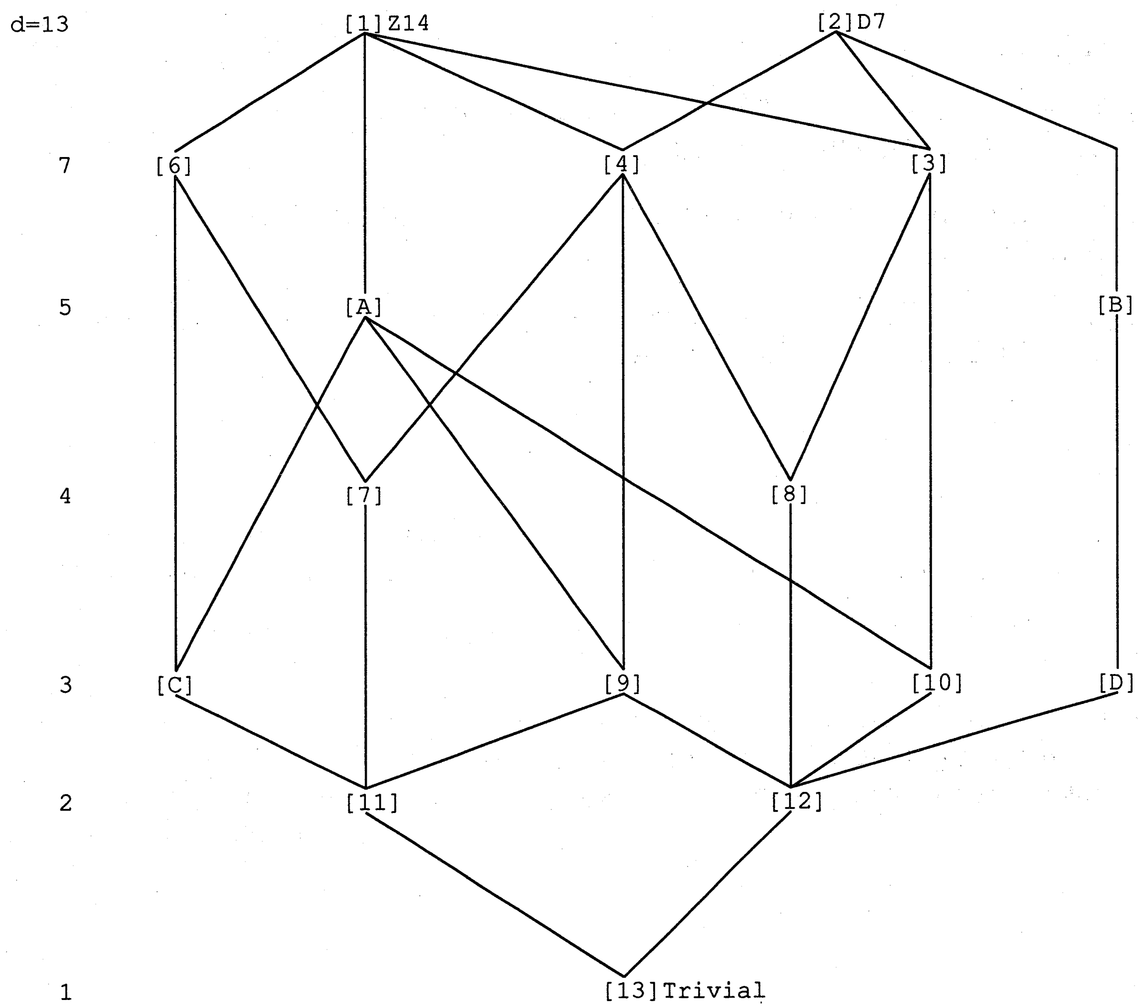
Sakita classified association schemes with 14 points in [5]. But there are some mistakes. Especially he forgot $1+1+3+3+3+3$ (valencies) cases. We correct his classification and obtain following.

```
d=14    [ 1]   cyclic group case (Z14)
        [ 2] dihedral group case  (D7)
   7    [ 3] {2,12,4,10,6,8,7+9+11+13+1+3+5} of Z14
        [ 4] {7,1+13,2+12,3+11,4+10,5+9,6+8}
        [ 6] {7,1+8,6+13,2+9,5+12,3+10,4+11}
   5    [ A] {7,2+4+8,6+10+12,9+11+1,13+3+5}
        [ B] Non circulant:{1+2+4,3+5+6,7,8+9+11,10+12+13} of D7
   4    [ 7] {7,1+8+6+13,2+9+5+12,3+10+4+11} of Z14
        [ 8] {2+12,4+10,6+8,7+9+11+13+1+3+5}
   3    [ 9] {7,2+4+6+8+10+12,9+11+13+1+3+5}
        [10] {2+4+8,6+10+12,7+9+11+13+1+3+5}
        [ C] {7,2+4+8+9+11+1,6+10+12+13+3+5}
        [ D] Non circulant:{1+2+3+4+5+6,7+8+9+11,10+12+13} of D7
   2    [11] {7,1+2+3+4+5+6+8+9+10+11+12+13} of Z14
        [12] {2+4+6+8+10+12,7+9+11+13+1+3+5}
   1    [13] Trivial scheme
```

where the relation matrix of [2] is the following.

$$\begin{bmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 13 & 12 & 11 & 10 & 9 & 8 \\
6 & 0 & 1 & 2 & 3 & 4 & 5 & 8 & 7 & 13 & 12 & 11 & 10 & 9 \\
5 & 6 & 0 & 1 & 2 & 3 & 4 & 9 & 8 & 7 & 13 & 12 & 11 & 10 \\
4 & 5 & 6 & 0 & 1 & 2 & 3 & 10 & 9 & 8 & 7 & 13 & 12 & 11 \\
3 & 4 & 5 & 6 & 0 & 1 & 2 & 11 & 10 & 9 & 8 & 7 & 13 & 12 \\
2 & 3 & 4 & 5 & 6 & 0 & 1 & 12 & 11 & 10 & 9 & 8 & 7 & 13 \\
1 & 2 & 3 & 4 & 5 & 6 & 0 & 13 & 12 & 11 & 10 & 9 & 8 & 7 \\
7 & 8 & 9 & 10 & 11 & 12 & 13 & 0 & 6 & 5 & 4 & 3 & 2 & 1 \\
13 & 7 & 8 & 9 & 10 & 11 & 12 & 1 & 0 & 6 & 5 & 4 & 3 & 2 \\
12 & 13 & 7 & 8 & 9 & 10 & 11 & 2 & 1 & 0 & 6 & 5 & 4 & 3 \\
11 & 12 & 13 & 7 & 8 & 9 & 10 & 3 & 2 & 1 & 0 & 6 & 5 & 4 \\
10 & 11 & 12 & 13 & 7 & 8 & 9 & 4 & 3 & 2 & 1 & 0 & 6 & 5 \\
9 & 10 & 11 & 12 & 13 & 7 & 8 & 5 & 4 & 3 & 2 & 1 & 0 & 6 \\
8 & 9 & 10 & 11 & 12 & 13 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0
\end{bmatrix}$$

Hesse diagram ($|X|$ =14)



References

[1] E.Bannai and T.Ito,Algebraic Combinatorics I:Association schemes, Benjamin/Cummings,Menlo Park, CA, 1984.

[2] A.E.Brouwer, A.M.Cohen and A.Neumaier,Distance Regular Graphs, Springer-Verlag,1989.

[3] E.Bannai, Subschemes of some association scheme, J.Algebra **144**(1991),167-188.

[4] E.Nomiyama, Classification of association schemes with at most ten vertices, Kyushu J. of Math.**49**(1995),163-195.

[5] Y.Sakita, Classification of association schemes with 14 vertices, master's thesis(1994).

[6] M.Hirasaka, Classification of association schemes with at 11 or 12 vertices, preprint

[7] E.Spence, Classification of Hadamard matrices of order 24 and 28, Discrete Math.**140**(1995),185-243.

[8] M.Hall.Jr., Hadamard matrices of order 16, J.P.L.Research Summary, **36**-10, Vol.1(1961),21-26.

[9] Sung Y.Song, Class 3 association schemes whose symmetrizations have two classes, J. of Comb.Thoery(A), **70**(1995),1-29.

Mitsugu Hirasaka and Yuji Suga
Graduate School of Mathematics , Kyushu University , Fukuoka 812 , JAPAN

If you link this URL, you can see results of the classfication of association schemes.
http://goldenfish.math.kyushu-u.ac.jp/Suga/COMB.htm
If you want to ask something about this Homepage, you send 'mail' to me, please.

suga@math.kyushu-u.ac.jp